## Math 102

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## Announcements

- Review Sessions for Thursday 10/25 Midterm
- Monday 10/22 in Buchanan A201, 3-7pm
- Tuesday 10/23 in CHBE 101, 3-7pm
- Bring questions if you have them!
- Midterm covers material up to and including Optimization.
- See the 'Midterm' tab on Canvas.


## Goals Today

- Data fitting
- Formula for nonzero $y$-intercept.
- Spreadsheet (demo)
- The Chain Rule


## Last Time - Best fit line



Points:
$\left(x_{1}, y_{1}\right),\left(x_{2}, y_{2}\right), \ldots,\left(x_{n}, y_{n}\right)$
Line: $y=a x$ for some $a$
Residuals: $i$-th residual is

$$
r_{i}=a x_{i}-y_{i}
$$

$$
S S R(a)=r_{1}^{2}+\ldots+r_{n}^{2}
$$

is minimized at $a=\frac{x_{1} y_{1}+\ldots+x_{n} y_{n}}{x_{1}^{2}+\ldots+x_{n}^{2}}$

## Line with Nonzero $y$-intercept

Points: $\left(x_{1}, y_{1}\right),\left(x_{2}, y_{2}\right), \ldots,\left(x_{n}, y_{n}\right)$
Line: $y=a x+b$ for some slope $a$, and $y$-intercept $b$
Residuals: $i$-th residual is

$$
r_{i}=a x_{i}+b-y_{i}
$$

$$
\begin{aligned}
\operatorname{SSR}(a, b) & =r_{1}^{2}+\ldots+r_{n}^{2} \\
& =\left(a x_{1}+b-y_{1}\right)^{2}+\ldots+\left(a x_{n}+b-y_{n}\right)^{2}
\end{aligned}
$$

Two variables: requires multivariable calculus!

## General best fit

- $x=$ Concentration of Penicillin
- $y=$ Bacterial survivors

| x | 0 | 2 | 3 | 4 | 5 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| y | 10000000 | 20000 | 1600 | 160 | 80 | 32 |

Question: Why would it be a bad idea to fit the data with a line?

## General best fit

Points: $\left(x_{1}, y_{1}\right),\left(x_{2}, y_{2}\right), \ldots,\left(x_{n}, y_{n}\right)$
Line: $y=f(x)$ for some function $f(x)$
Residuals: $i$-th residual is

$$
r_{i}=f\left(x_{i}\right)-y_{i}
$$

$$
\begin{gathered}
S S R=r_{1}^{2}+\ldots+r_{n}^{2} \\
=\left(f\left(x_{1}\right)-y_{1}\right)^{2}+\ldots+\left(f\left(x_{n}\right)-y_{n}\right)^{2}
\end{gathered}
$$

(Spreadsheet demo)

## The Chain Rule

Example: Calculate the derivative.

$$
h(x)=\sqrt{x^{5}+11}
$$

